**Quadratics (1)**

**1.**  **(a)** State the relation between such that the equation ( has equal roots.

**(b)** If the equation ( has equal roots,

show that the roots for the equation ( are equal.

**(a)** The condition for the equation has equal roots is

**(b)** If the equation has equal roots,

**Method 1**

For the equation

Since

Therefore has equal roots.

**Method 2**

Since

(equal roots)

**2.** If x is real, show that the expression does not have a value between .

Since x is real,

**3.**  Let the equations and ( have one non-zero common root.

Form an equation with the other roots of these equations.

Let be the non-zero common root. Then by Vieta Theorem the other roots are .

Also and and their difference is

giving (check that )

Thus, the other roots are and .

Sum of roots =

Product of roots =

Therefore the required equation is

**4.** If are real numbers, show that the roots of the equation is real.

If one of the roots is twice the other, show that .

For the equation

Hence, the roots of the equation is real.

If one of the roots is twice the other, let be the roots.

By Vieta Theorem,

…(2)

From (1),

From (2),

**5.** If are the roots of the equation and are the roots of the equation , show that :

**(a)** If , then ,

**(b)** If , then .

**(a)** By Vieta Theorem, and

Since

**(b)**

**6.** If the equation ( has real roots, show that the equation

has also real roots.

Show that if and are the roots for the first equation, then the product of roots of the second equation is

The equation ( has real roots if and only if

For the equation ,

Therefore the second equation has real roots.

For the first equation, by Vieta Theorem, ,

The product of roots of the second equation is

**7.** If are roots of the equation and are roots of the equation .

Express in terms of .

By Vieta Theorem, ,

=

**8.** Show that the expression is positive and find its greatest value.

Hence find the smallest values of .

Sketch the functions of together on the same graph.

Therefore is positive.



**Yue Kwok Choy**

**4/9/2016**